Using Baumgarte's Method for Index Reduction in Modelica

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Abstract

Modeling and simulation of some types of constrained mechanical systems, such as closed kinematic chains, can be challenging in the Modelica language. One reason is because component-oriented modeling for such systems results in a set of high-index nonlinear differential algebraic equations (DAEs). Modelica compliers, such as Dymola, use the method of "dummy derivatives" to reduce the index automatically, for very good and fundamental reasons. However, for closed chains it has some disadvantages, and there are other methods which have advantages especially for consistent initialization and use cases beside simulation, such as control system design.

In this paper we show by example how Baumgarte's method of index reduction can be used in Modelica to reduce the index of a constrained mechanical system *prior* to compilation. The method is applied at modeling time, and results in a model that is of lower index to begin with, so that the compiler does not have to reduce further the index. We begin by showing Baumgarte's method for the simple pendulum. The method and its properties can be understood from this example. But our primary target is a delta robot, for which we derive a singularity-free, index 1 differential-algebraic equation model. No automatic index reduction is done at compile time, and no dynamic state selection is required at simulation time. We find that the method is amenable to Modelica's object oriented modeling paradigm, and results in simulation code that can be, at least anecdotally, faster. We construct several components of a feedback controller directly from the index-1 system model, and show how consistent initial conditions can be computed in this formulation.

We provide a third example, elevator cable sway, in which the method is vital to simulation and feedback control system design. Indeed we have failed to construct a model of this system using the Modelica Standard Library and Dymola's built-in index reduction algorithms. However using Baumgarte's method to essentially prereduce the index results in a simple model. We have developed several feedback control strategies for reducing cable sway that occurs during events such as earthquakes.

The method can be extended to model some situations in which constraints are time-varying, such as in loss-of-contact or constraint-breaking problems. More generally, we believe the method may find successful application in other domains, particularly for problems in which consistent initial conditions are difficult to compute.

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