# An Extended Luenberger Observer for HVAC Application using FMI

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## Abstract

State estimation is one of the important use cases for the Functional Mockup Interface (FMI). For example, states of a nonlinear continuous-time model can be estimated from discrete-time measurements of the input and output of a plant using a continuous-discrete Extended Kalman Filter (EKF), realized using the co-simulation form of a Functional Mockup Unit (FMU) of the plant. Fundamentally, the EKF, and its various extensions estimate the state in a two-step process. In the prediction step, the EKF computes the predicted state estimate using a discretized plant model. Then in the correction step, the covariance and gain are computed as a function of the predicted state estimate, and the predicted state estimate state is corrected. The discrete-time prediction model is then initialized using the corrected state, and the process is repeated.

An *observer* is an alternative technology for estimation of the plant states and parameters. An observer is a deterministic, continuous-time dynamical system that takes as input the measured input and measured output of the plant, and produces as its output an estimate of the state of the plant. It is similar to the Kalman filter, but based on deterministic assumptions and mathematics. Fundamentally, the concept of *output injection* is used to stabilize the observer error dynamics, which govern the difference between the estimated state and the plant state. Output Injection means that a signal is injected (added) to the derivative of the observer state vector as stabilizing feedback. Because of this, it is the continuous-time dynamics of the plant *with* output injection that needs to be simulated as a continuous-time feedback system. There are not separate, algorithmic prediction and correction steps.

In this paper we show how an instantiation of a model-exchange type of FMU can be used with the Dymola tool to realize output injection, enabling design and implementation of linear and nonlinear state observers and specifically the Extended Luenberger Observer (ELO). Our specific interest is to estimate unmeasured performance variables of a building and HVAC system as a part of a building "digital twin." Toward this end we have considered several alternative methods to estimate the performance variables, including various flavors of the EKF. However, these may prove too computationally burdensome for our application because the number of states can be large (hundreds), the number of measurements can be large (tens to hundreds), and the EKF can be computationally challenging because of the covariance update, although there are many techniques such as model reduction and square root filtering that are available to improve its computational efficiency. More importantly, an EKF can fail to converge, or in some cases, cause the model to fail at run time, at least for our building HVAC applications. Convergence failures are caused by some of the characteristics of the model that we consider in this paper, which are not unusual for this field of application. The model is stiff (with time constants ranging from milliseconds to several weeks — eight orders of magnitude), and is numerically ill-conditioned (with states varying 8-9 orders of magnitude because of the choice of units). Thus the Jacobian may not accurately predict the state over the fixed and usually large EKF sample time, causing it to diverge. Moreover, the model itself contains state constraints, such as a non-negative limit on mass concentrations, which can be violated at run time because of the EKF correction step, causing a run-time error.

On the other hand, the ELO is relatively simple and light-weight computationally. In its simplest form, it uses a constant feedback gain matrix that is computed at design time from the steady-state solution of a Ricatti equation, and therefore avoids the real-time covariance update and computation of the system Jacobian that is necessary for the EKF. Further, it may offer improved stability and performance advantages over the EKF (and similar filters) for certain applications because it makes use of implicit variable-step solvers for the continuous-time model.

We present the design steps required to compute the feedback gain from a linearization of the model. The key step is to compute a sequence of model order reductions which remove weakly observable and controllable modes from the model. We show how the feedback is realized in a Modelica model that uses an FMU for model exchange. We close by making several remarks concerning the design of discrete-time state estimators and some of the needs for dynamic analysis of building models.
*Keywords: Estimation, Buildings, HVAC, FMI, FMU*